Measured difference between ^{206}Pb, ^{205}Tl charge distributions and the proton $3s_{1/2}$ wave function

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The difference between the charge distributions of ^{206}Pb and ^{205}Tl, $\Delta\rho_c$, was measured many years ago [1, 2]. It offers a good opportunity to study possible effects of short range correlations on the shell model wave function of a proton in the $3s_{1/2}$ orbit. Effects of this kind were estimated by comparing measured cross sections of various reactions to those calculated using shell model wave functions. Usually, the measured values were lower than the calculated ones. The difference between the charge distributions considered here cannot be depleted. The integrated difference must be exactly equal to the charge difference between the two isotones, 1 proton charge. The effects of short range correlations in this case can only change the *shape* of the difference between the charge distributions.

The corresponding single particle potential for a nucleon is

$$V_{cen}(r) = E + \frac{\hbar^2}{2m}S(r) - V_{coul}(r), \qquad S(r) = \frac{d^2R}{dr^2}\frac{1}{R(r)}, \tag{1}$$

where $V_{coul}(r)$ is the Coulomb potential and $R(r)$ is related by $\psi_{nlj}(\vec{r}) = \frac{R_{nlj}(r)}{r}Y_{lj}$.

The single particle radial density $\rho(r)$ is related to the square of the radial wave function R by

$$R^2(r) = 4\pi r^2\rho(r)\ . \tag{2}$$

From (2) it is possible to extract the wave function $R(r)$ and use Eq. (1) to deduce the corresponding single particle potential, but this leads to numerical complications. Therefore, we developed a method to determine the potential directly from the density and its first and second derivatives. Using the Schrodinger equation for the radial wave-function $R(r)$, we obtain the simple relations

$$S(r) = \frac{1}{2R^2}\left[\frac{d^2(R^2)}{dr^2} - \frac{1}{2}\left[\frac{1}{R}\frac{d(R^2)}{dr}\right]^2\right], \tag{3}$$

and

$$S(r) = \frac{1}{2\rho}\left[\frac{d^2\rho}{dr^2} + \frac{2}{r}\frac{d\rho}{dr} - \frac{1}{2\rho}\left(\frac{d\rho}{dr}\right)^2\right]. \tag{4}$$

The experimental values of that difference have features which are very similar to those due to the wave function of a proton in a $3s_{1/2}$ orbit. Within the experimental error bars, all its values are non-negative and there are two zero values for $r > 0$ which correspond to the two nodes of the $3s$ wave function $R_p(r)/r$. If the point proton distribution $\Delta\rho_p(r)$ is due to a $3s$ wave function, two more

conditions should be satisfied, in addition to its having first and second derivatives for all r values. At r where $\Delta\rho_p(r) = 0$, also its first derivative should vanish. The condition that if $R_p(r) = 0$, also its second derivative must vanish at that r leads to the other condition. Where the point proton distribution $\Delta\rho_p(r) = 0$, the corresponding expression in the square brackets on the right hand side of (4) should vanish. These conditions are necessary for deriving the single particle potential V, using Eqs. (1) and (3) or (4), from a parametrized point proton distribution $\Delta\rho_p(r)$ fitted to the experimental data. It is difficult to see whether these conditions are satisfied by the measured difference of charge distributions. The experimental accuracy is not sufficient, especially near the zero values.

We started by deriving and employing a new relation [3] between the potential V and the single particle density and its first and second derivatives, Eqs. (1), (3) and (4). Around its minima, the experimental uncertainty in $\Delta\rho_c(r)$ is larger than its value. Hence, no reliable potential can be obtained.

In view of this situation, we tried to construct nuclear single particle potentials V whose proton $3s_{1/2}$ orbit in ^{206}Pb yield charge distributions which best fit the electron scattering data. We found several potentials which yield fair fits to the data (Fig. 1). The fair agreement with fitted potentials may be an

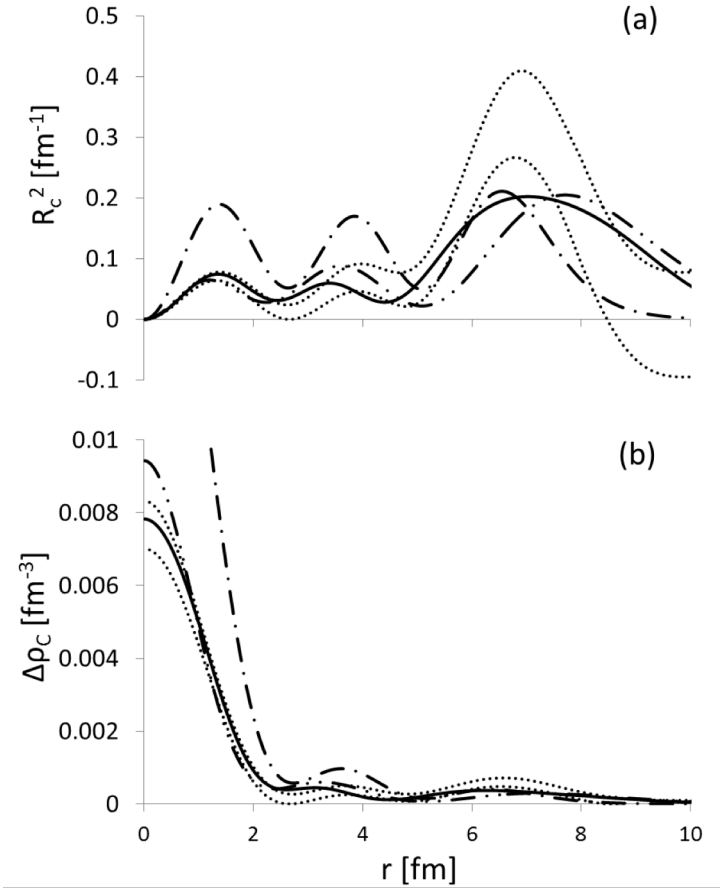

FIG. 1. Experimental values of $\boldsymbol{R_c^2(r) = 4\pi r^2 \Delta\rho_c(r)}$ (a) and $\boldsymbol{\Delta\rho_c(r)}$ (b) plotted between dotted lines of error limits. They are compared to calculated charge distributions due to the $3s_{1/2}$ wave functions of the fitted $V_F(r)$ potential (solid lines), the fitted Wood-Saxon $V_{FWS}(r)$ potential (double dotted-dashed lines) and the conventional $V_{WS}(r)$ potential (dashed-dotted lines.

indication that effects of short range correlations on charge distributions due to shell model wave functions are not significant. More accurate experimental data for $\Delta\rho_c(r)$ with uncertainty smaller by a factor of two or more may answer the question how well can the data be reproduced by a calculated $3s_{1/2}$ wave function.

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